Unified Method for Determining the Complex Propagation Constant of Reflecting and Nonreflecting Transmission Lines

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***Abstract*—In this letter, a unified method for computing the complex propagation constant γ of reflecting and nonreflecting lines is presented. The method uses a new matrix representation of the wave cascade matrix of a line having any characteristic impedance. To overcome the sign ambiguity problem inherent to the classical method some parameters of the fictitious $\mathbf{X}_{\mathrm{AM}}$ matrix are used and determined by the method itself. The success of the new procedure to resolve the sign ambiguity problem lies in the new matrix representation of the wave cascade matrix of a line having any characteristic impedance and in the reliable criterion to determine the a_m/c_m and b_m parameters of the fictitious $\mathbf{X}_{\mathrm{AM}}$ matrix.**

***Index Terms*—ABCD matrix, eigenvalues, wave cascade matrix, wave propagation constant.**

I. Introduction

USUALLY, the complex propagation constant γ is determined from scattering parameters measurements (S_{ij}) performed on two lines (Line–Line Method) having the same characteristic impedance Z_L but different lengths [1]. Once the S_{ij} parameters are measured either the ABCD [2] or wave cascading matrix (WCM) [3]–[5] may be used for of γ determination. Therefore the first step in the broadband γ calculation is the determination of the eigenvalues λ_1 and λ_2 [3], [5]. Different expressions for computing λ_2 and λ_1, using the L-L method along with ABCD and WCM, are shown in Table I. On the other hand, experimental results have shown that neither λ_1 nor λ_2 represents in broadband a continuous wave in phase and magnitude. Indeed, in nonreflecting lines λ_1 and λ_2 exhibit discontinuities in phase and magnitude located at the vicinity of 90° and 270° and the reflective lines λ_1 and λ_2 may exhibit additional discontinuities in phase and magnitude in the vicinity of 180° and 360°. Briefly stated, to date the main problem with the broadband λ determination when using the ABCD or WCM is the lack of a reliable criterion to discern between the two eigenvalues λ_1 and λ_2, a continuous wave in phase and magnitude λ representing either an incident or reflected wave. In the case of nonreflecting lines a reliable method for determining a continuous λ was already developed [6]. In this work, a unified method for determining λ and γ of reflecting and nonreflecting lines is presented. The method is based on a new matrix representation of the WCM of a line having any characteristic impedance Z_L.

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TABLE I
COMPARISON OF DIFFERENT METHODS FOR EIGENVALUES COMPUTATION USING ABCD AND WCM FORMALISM

	Method using similar matrix properties $trace(A)=\sum_{1}^{2} a_{ii}$	Method using determinants properties $\det(A)=a_{11}a_{22}-a_{12}a_{21}$
Using chain $M_{1,2}^{A}$ ABCD matrix formalism	$\lambda_{1,2}^{AS}=\frac{trace(\mathbf{M}_2^A\mathbf{M}_1^{A^{-1}})\pm\sqrt{\left[trace(\mathbf{M}_2^A\mathbf{M}_1^{A^{-1}})\right]^2-4}}{2}$	$\lambda_{1,2}^{AD}=\frac{-\left[2-\frac{\det(\mathbf{M}_1^A+\mathbf{M}_2^A)}{\det(\mathbf{M}_2^A)}\right]\pm\sqrt{\left[2-\frac{\det(\mathbf{M}_1^A+\mathbf{M}_2^A)}{\det(\mathbf{M}_2^A)}\right]^2-4}}{2}$
Using Wave cascade matrix $M_{1,2}^{T}$ WCM formalism	$\lambda_{1,2}^{TS}=\frac{trace(\mathbf{M}_2^T\mathbf{M}_1^{T^{-1}})\pm\sqrt{\left[trace(\mathbf{M}_2^T\mathbf{M}_1^{T^{-1}})\right]^2-4}}{2}$	$\lambda_{1,2}^{TD}=\frac{-\left[2-\frac{\det(\mathbf{M}_1^T+\mathbf{M}_2^T)}{\det(\mathbf{M}_2^T)}\right]\pm\sqrt{\left[2-\frac{\det(\mathbf{M}_1^T+\mathbf{M}_2^T)}{\det(\mathbf{M}_2^T)}\right]^2-4}}{2}$

II. Method Description

The implementation of the method requires two lines having the same characteristic impedance but different length as shown in Fig. 1. The two ports referred to as X and Y correspond to transitions used for ensuring the connection between the lines and the vector network analyzer at the line input and output ports. X and Y include the microwave probes, coaxial to microstrip microwave connectors (launchers) and the necessary hardware for the network analyzer. We use the matrices, $\mathbf{T_A}$, $\mathbf{T_B}$, $\mathbf{T_{L1}}$ and $\mathbf{T_{L2}}$, for modeling respectively, the transitions X, Y and the line L_1 and L_2. Next, we assume that X and Y are unequal. The measured scattering parameters S, expressed in WCM matrix form by $\mathbf{M_1}$ and $\mathbf{M_2}$ of the complete structures, can be expressed in the form

$$\mathbf{M_1} = \mathbf{T_A}\mathbf{T_{L1}}\mathbf{T_B} \tag{1}$$

$$\mathbf{M_2} = \mathbf{T_A}\mathbf{T_{L2}}\mathbf{T_B}. \tag{2}$$

Combining (1) and (2), the matrix product $\mathbf{M_1}\mathbf{M_2^{-1}}$ is given by

$$\mathbf{M_1}\mathbf{M_2^{-1}} = \mathbf{T_A}\mathbf{T_{L_1}}\mathbf{T_{L_2}^{-1}}\mathbf{T_A^{-1}}. \tag{3}$$

From the matrix theory it should be noticed that $\mathbf{M_1}\mathbf{M_2^{-1}}$ and $\mathbf{T_{L_1}}\mathbf{T_{L_2}^{-1}}$ are similar matrices. In fact, properties of similar matrices have been used in the classical method to determine the eigenvalues λ_1 and λ_2 of the matrix product $\mathbf{M_1}\mathbf{M_2^{-1}}$.

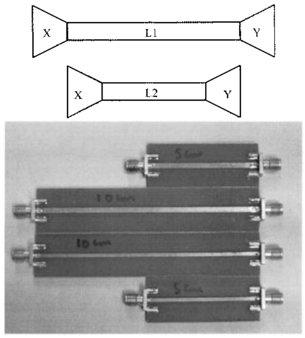


Fig. 1. Structures used for the implementation of the Line–Line method.

The WCM matrix $\mathbf{T_{Li}}$ of a line of length l_i ($i = 1, 2$) featuring the unknown characteristic impedance Z_L can be expressed in the form

$$\mathbf{T_{Li}} = \frac{1}{(1-\Gamma^2)\, e^{-\gamma l_i}} \begin{pmatrix} e^{-2\gamma l_i} - \Gamma^2 & \Gamma\left(1 - e^{-2\gamma l_i}\right) \\ -\Gamma\left(1 - e^{-2\gamma l_i}\right) & 1 - \Gamma^2 e^{-2\gamma l_i} \end{pmatrix} \tag{4}$$

where $\Gamma = Z_L - Z_o / Z_L + Z_o$ *and* Z_o *is the reference impedance*.

The novelty of the method for computing λ of lines of unknown impedance is based on the fact that expression (4) may be expressed as

$$\mathbf{T_{Li}} = \mathbf{T}_\Gamma \mathbf{T_{50Li}} \mathbf{T}_\Gamma^{-1} \tag{5}$$

where

$$T_\Gamma = \begin{pmatrix} 1 & \Gamma \\ \Gamma & 1 \end{pmatrix} \tag{6}$$

$$\mathbf{T_{50Li}} = \begin{pmatrix} e^{-\gamma li} & 0 \\ 0 & e^{\gamma li} \end{pmatrix}. \tag{7}$$

An expression similar to (5) for impedance transforming between ports was published in [7]. The main difference between [7, Eqs. 5 and 91] lies on the fact that (5) does not represent a change of reference impedance of the line. The new WCM representation of a line having any characteristic impedance, modeled with (5), suggests that any transmission line having any characteristic impedance can be modeled as a nonreflecting line embedded in two symmetrical and reciprocal transitions in our case modeled by the T_Γ matrix.

Using (5), (3) becomes

$$\mathbf{M_1 M_2^{-1}} = \mathbf{X_{AM} T_{50L1} T_{50L2}^{-1} X_{AM}^{-1}} \tag{8}$$

where

$$\mathbf{X_{AM}} = \mathbf{T_A} \mathbf{T}_\Gamma \tag{9}$$

$$\mathbf{X_{AM}} = \begin{pmatrix} \mathbf{r_{11}} & \mathbf{r_{12}} \\ \mathbf{r_{21}} & \mathbf{r_{22}} \end{pmatrix} = \mathbf{r_{22}} \begin{pmatrix} \mathbf{a_m} & \mathbf{b_m} \\ \mathbf{c_m} & \mathbf{1} \end{pmatrix}. \tag{10}$$

Defining the matrix product $\mathbf{M_1 M_2^{-1}}$ by

$$\mathbf{T} = \mathbf{M_1 M_2^{-1}} = \begin{pmatrix} \mathbf{t_{11}} & \mathbf{t_{12}} \\ \mathbf{t_{21}} & \mathbf{t_{22}} \end{pmatrix} \tag{11}$$

and the matrix product $\mathbf{T_{50L1} T_{50L2}^{-1}}$ as

$$\mathbf{T_{X50}} = \mathbf{T_{50L1} T_{50L2}^{-1}} = \begin{pmatrix} \lambda & \mathbf{0} \\ \mathbf{0} & \frac{\mathbf{1}}{\lambda} \end{pmatrix}; \lambda \in C \tag{12}$$

where

$$\lambda = e^{\gamma(l_2 - l_1)} \tag{13}$$

the expression (8) becomes

$$\mathbf{T} = \mathbf{X_{AM} T_{X50} X_{AM}^{-1}}. \tag{14}$$

Finally, $\mathbf{T_{X50}}$ may be written as

$$\mathbf{T_{X50}} = \mathbf{X_{AM}^{-1} T X_{AM}}. \tag{15}$$

Using (10), (11), and (13), (15) becomes (16) as shown at the bottom of the page.

Comparing each term of the matrices on both sides of (16), λ may be expressed in the form by

$$\lambda = \frac{\left[\frac{\mathbf{a_m}}{\mathbf{c_m}}\mathbf{t_{11}} - \mathbf{b_m}\frac{\mathbf{a_m}}{\mathbf{c_m}}\mathbf{t_{21}} - \mathbf{b_m t_{22}} + \mathbf{t_{12}}\right]}{\frac{\mathbf{a_m}}{\mathbf{c_m}} - \mathbf{b_m}} = \frac{\frac{\mathbf{a_m}}{\mathbf{c_m}} - \mathbf{b_m}}{\frac{\mathbf{a_m}}{\mathbf{c_m}}\mathbf{t_{22}} + \mathbf{b_m}\frac{\mathbf{a_m}}{\mathbf{c_m}}\mathbf{t_{21}} - \mathbf{b_m t_{11}} - \mathbf{t_{12}}} \tag{17}$$

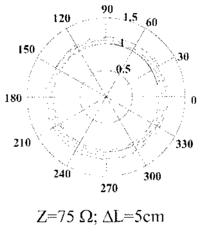
and

$$\mathbf{b_m^2 t_{21}} + \mathbf{b_m}(\mathbf{t_{22}} - \mathbf{t_{11}}) - \mathbf{t_{12}} = \mathbf{0} \tag{18}$$

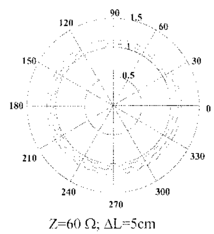
$$\left(\frac{\mathbf{a_m}}{\mathbf{c_m}}\right)^{\mathbf{2}} \mathbf{t_{21}} + \frac{\mathbf{a_m}}{\mathbf{c_m}}(\mathbf{t_{22}} - \mathbf{t_{11}}) - \mathbf{t_{12}} = \mathbf{0}. \tag{19}$$

It is interesting to comment on the main features of the classical and the new methods for λ determination. In both methods sign ambiguity problems exist. In the classical method as already mentioned the problem is to determine a continuous λ from the previous knowledge of the two discontinuous eigenvalues λ_1 and λ_2. Unfortunately, in spite of previous efforts [5], [8], [9] there does not exist a reliable criterion to determine a continuous λ using the procedures indicated in Table I. As for the new method, the sign ambiguity is indirect and is present in the solution of the quadratic equations (18)–(19). Fortunately, a reliable criterion to resolve the sign ambiguity in the solution of (18)–(19) was already established [10]. In summary, it seems that the success for determining a continuous λ lies in the new matrix representation of the wave cascade matrix of a line having any characteristic impedance and in the reliable criterion to compute a_m/c_m and b_m.

$$\begin{pmatrix} \lambda & \mathbf{0} \\ \mathbf{0} & \frac{\mathbf{1}}{\lambda} \end{pmatrix} = \frac{\mathbf{1}}{\mathbf{a_m} - \mathbf{b_m c_m}} \begin{pmatrix} \mathbf{c_m}\left[\frac{\mathbf{a_m}}{\mathbf{c_m}}\mathbf{t_{11}} - \mathbf{b_m}\frac{\mathbf{a_m}}{\mathbf{c_m}}\mathbf{t_{21}} - \mathbf{b_m t_{22}} + \mathbf{t_{12}}\right] & -\left[\mathbf{b_m^2 t_{21}} + \mathbf{b_m}(\mathbf{t_{22}} - \mathbf{t_{11}}) - \mathbf{t_{12}}\right] \\ \mathbf{c_m^2}\left[\left(\frac{\mathbf{a_m}}{\mathbf{c_m}}\right)^{\mathbf{2}}\mathbf{t_{21}} + \left(\frac{\mathbf{a_m}}{\mathbf{c_m}}\right)(\mathbf{t_{22}} - \mathbf{t_{11}}) - \mathbf{t_{12}}\right] & \mathbf{c_m}\left[\frac{\mathbf{a_m}}{\mathbf{c_m}}\mathbf{t_{22}} + \mathbf{b_m}\frac{\mathbf{a_m}}{\mathbf{c_m}}\mathbf{t_{21}} - \mathbf{b_m t_{11}} - \mathbf{t_{12}}\right] \end{pmatrix} \tag{16}$$

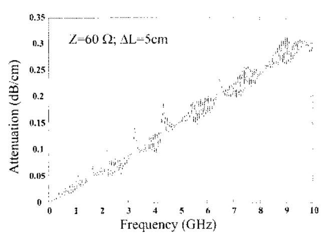


(a)

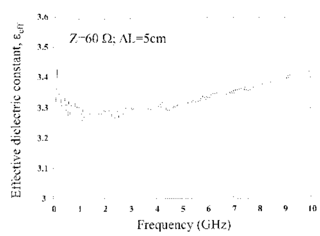


(b)

Fig. 2. Polar plot of the traveling wave λ illustrating a continuous behavior in phase and in the frequency range of 0.040–10 GHz: (a) $Z_L = 60\ \Omega$ and (b) $Z_L = 75\ \Omega$.



(a)



(b)

Fig. 3. (a) Attenuation per physical length versus frequency and (b) effective dielectric constant versus frequency for a microstrip lines computed with the new technique.

III. Experimental Results

In order to demonstrate the usefulness of the proposed technique, microstrip lines fabricated on TFR4 have been investigated. Scattering parameters of these lines have been measured using a HP8510C system in the frequency range 45 MHz–10 GHz without any previous calibrations. Then the measured S parameters of the microstrip lines are converted to WCM to determine λ. Fig. 2 shows a polar plot of λ determined using (17). It should be noted that continuous phase and magnitude variations throughout the frequency band are observed in Fig. 2. The continuous phase and magnitude variations of λ allow the broadband determinations of the phase shift of lines and the line losses. Once λ is calculated the wave propagation constant γ is computed using the expression given by

$$\gamma = \frac{1}{L_2 - L_1} \ln \lambda. \qquad (20)$$

The wave propagation constant γ is directly related to the attenuation coefficient and phase constant α and β respectively ($\gamma = \alpha + j\beta$; $\beta = 2\pi f\sqrt{\varepsilon_{eff}}/c$; where : f is the frequency and c is the light velocity). Finally, the attenuation coefficient α per centimeter length and the effective dielectric constant ε_{eff} have been determined and their variations versus frequency are shown in Fig. 3.

IV. Conclusions

A unified method for determining λ and γ of reflecting and nonreflecting transmission lines has been presented. The unified method is based on the new WCM expression for modeling transmission lines for an arbitrary characteristic impedance. Furthermore, the new WCM expression indicates that any homogeneous transmission line may be modeled as a nonreflecting line embedded in two symmetrical and reciprocal transitions. The usefulness of the unified method for determining λ and γ of reflecting and nonreflecting transmission lines has been demonstrated by evaluating the attenuation constant α and the effective dielectric constant ε_{eff} of reflecting microstrip lines fabricated on TFR4 substrates.

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